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Qualitative Analysis of Critical Points

More Precise Definition of Stability (after Dirichlet)

"The equilibrium [of a mechanical system] is stable if, in displacing the points of the system from their equilibrium positions by an infinitesimal amount and giving each one a small initial velocity, the displacements of different points of the system remain, throughout the course of the motion, contained within small prescribed limits"

Consequences of Dirichlet's Definition

(1) Stability is a quality of each equilibrium solution (We cannot generally state *a priori* **that a system, as a whole, is stable or unstable)**

(2) The problem of ascertaining the stability of a solution concerns the neighborhood of that solution and is therefore a local one

(3) The concept of stability is inherently dynamic in nature. But for conservative systems, dynamics can be "factored out" of the problem, and we are left with static criteria

Goal of Chapter

Qualitative classification of bifurcation and limit points as regards global & post critical behavior - focus on energetic interpretation resembling rolling ball

No equations - all pictures

Assumptions & Limitations

In this and following 2 Chapters we will not neglect changes in geometry

However, we still confine attention to conservative systems

Their stability can be analyzed by static criteria, in particular the tangent stiffness spectrum

General Procedure

Run a geometrically nonlinear analysis, getting the control vs state response

Look for critical points but keep going (traverse them)

Examine the spectrum of the tangent stiffness matrix as CPs are traversed

Classification of EquilibriumConfigurations as per Spectrum of Tangent Stiffness K There

If K evaluated at an equilibrium position is	***The potential energy*** Π ***at that position has a***	***Then the equilibrium position is***
positive definite	**strict minimum**	**stable**
positive semidefinite*	**cylindrical or inflexion point**	**neutrally stable**
indefinite	**saddle point**	**unstable**

* also known as nonnegative

It is sufficient to apply this test at just one point between CPs to declare the whole path stable or unstable - Why?

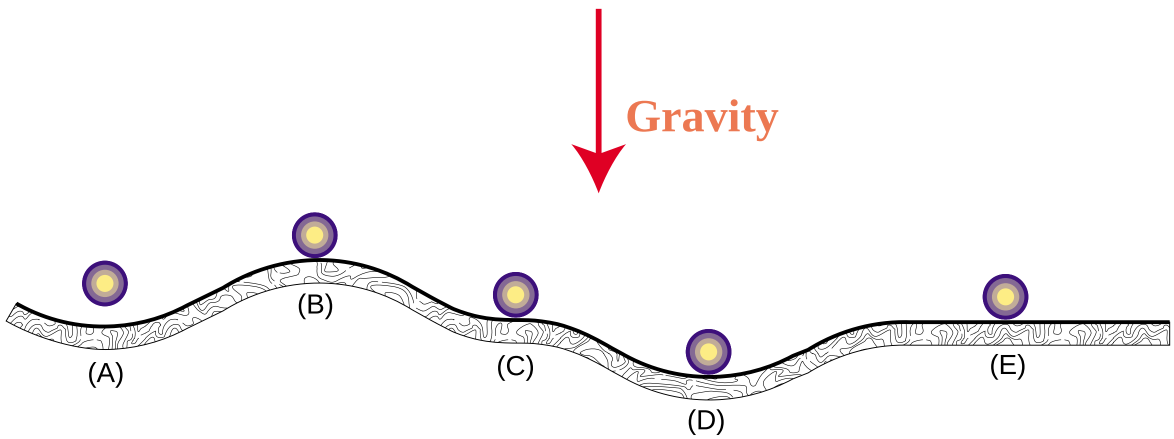
Qualitative Approach to Stability

Instead of mathematics we can take a qualitative look at stability loss at various types of critical points using a physical analogy:

A "rolling ball" over ground under action of gravity, mapped to a potential energy function interpretation

Several examples of applications of this visualization technique will be given

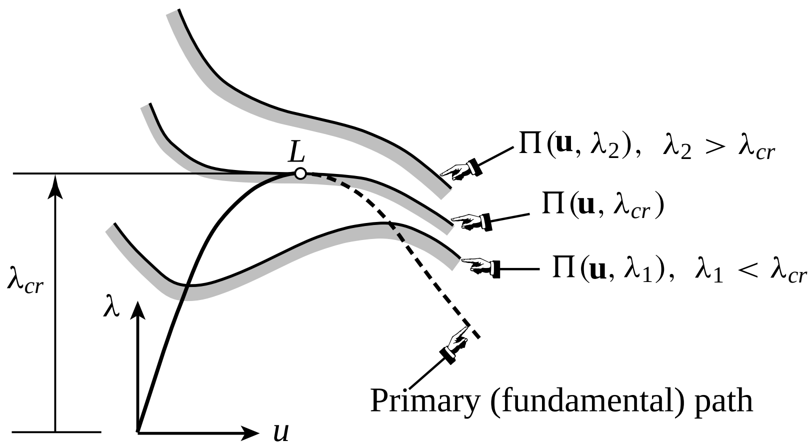
Recall the Picture for the Little Kids



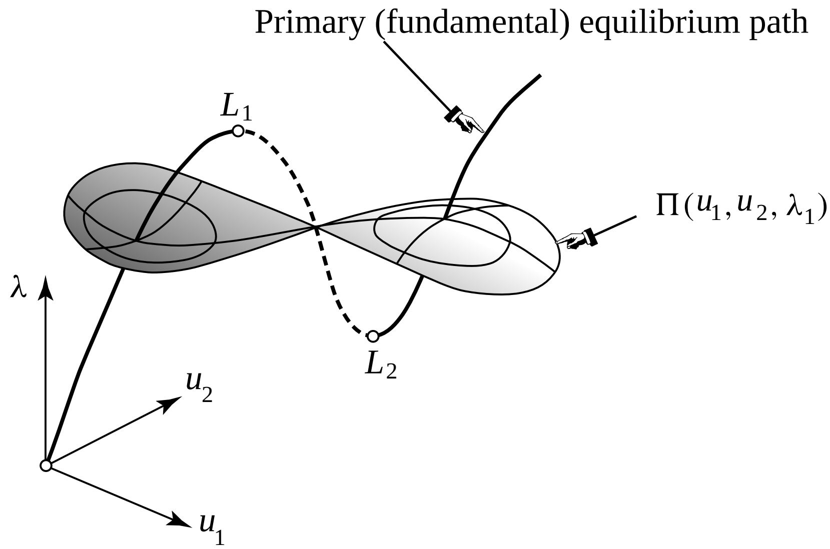
For the grown ups we map

Gravity → Potential Energy

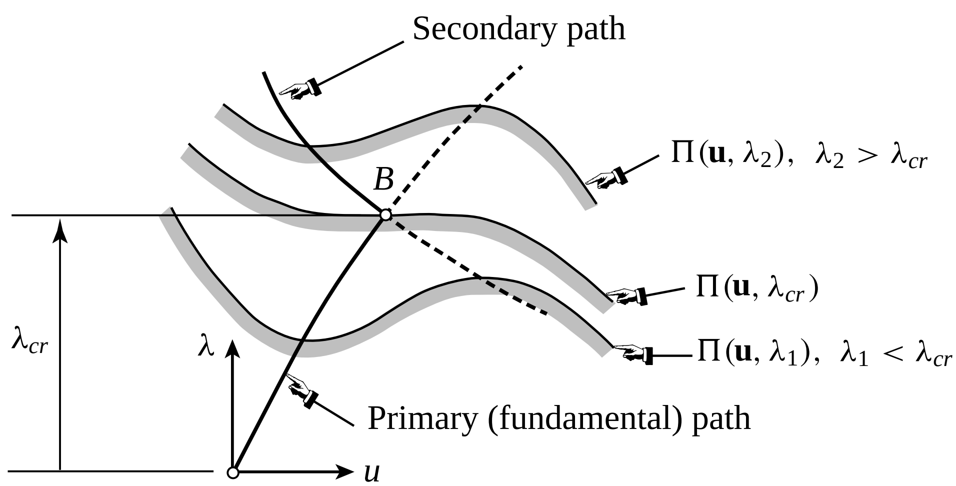
Stability Exchange at a Limit Point (1 DOF)



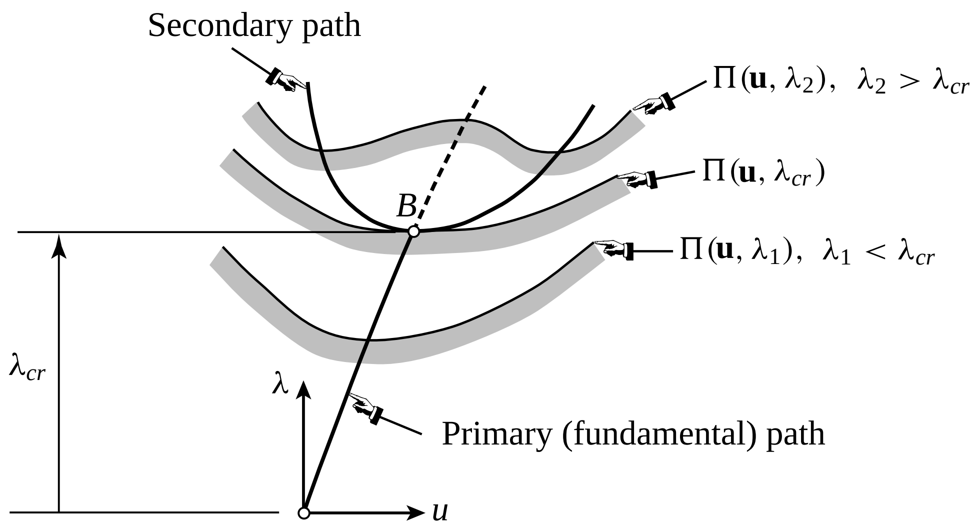
Stability Exchange at a Limit Point (2 DOF)



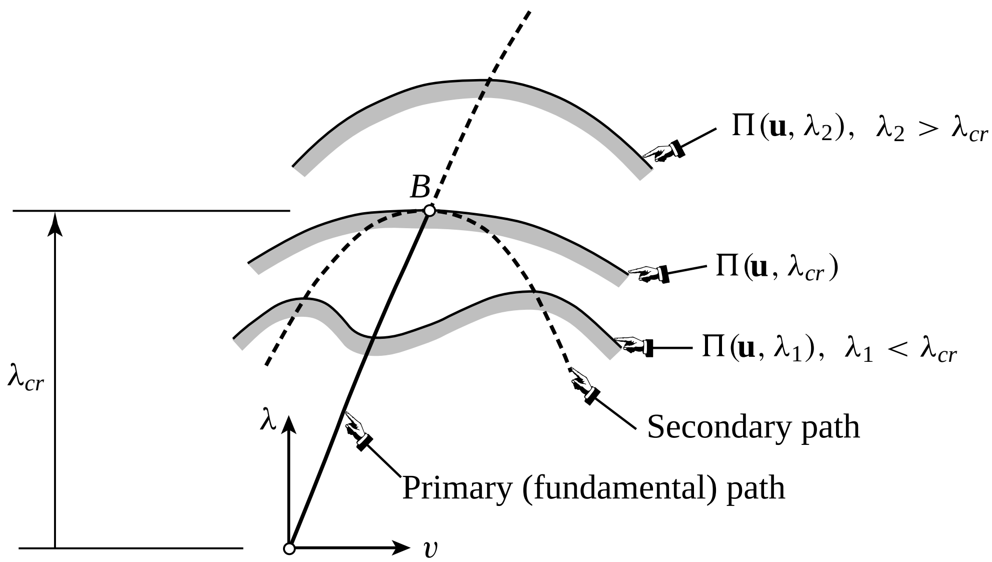
Asymmetric Bifurcation



Stable-Symmetric Bifurcation



Unstable-Symmetric Bifurcation



Next Two Chapters Deal With the Mathematical Models

Chapter 33: nonlinear stability analysis levels

Chapter 34: effect of imperfections