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Linearized Prebuckling: Limitations

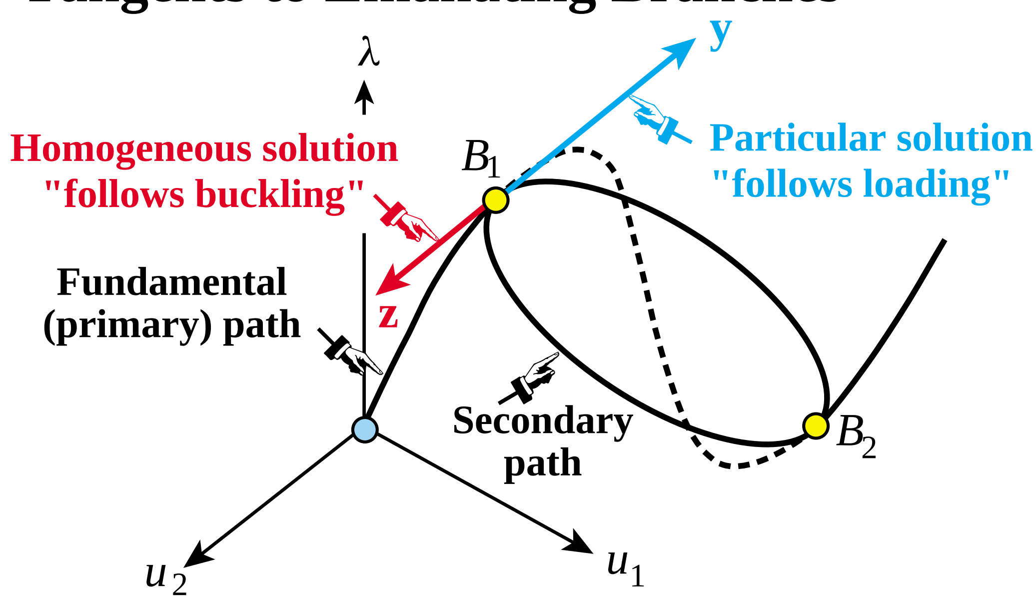
Bifurcation Point Analysis Levels

Level 0: Critical point location <- Chapter 5

Level 1: Tangents to emanating branches <- This Chapter

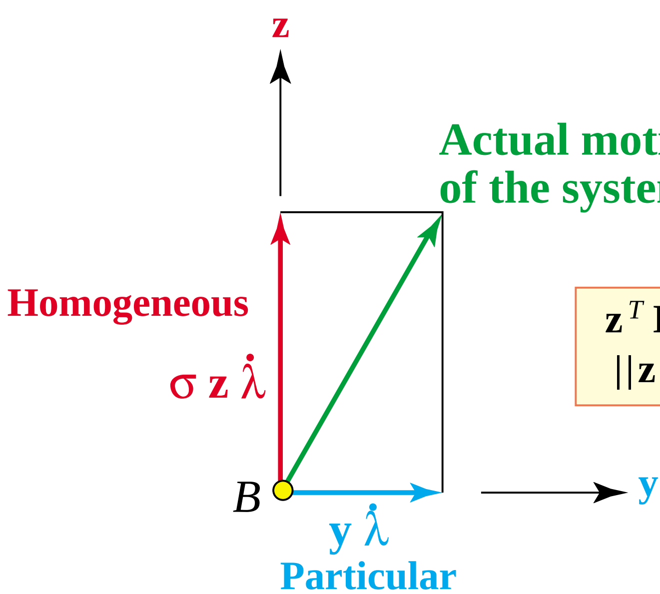
We will only consider isolated bifurcation points

State Decomposition At Isolated Bifurcation Point: Tangents to Emanating Branches



Vectors Ky and z are mutually orthogonal (proved later)
Together they define a plane in control-state space.
All important physics happens in that plane, regardless of how many DOF the model has: 2 or a million

Drawing the State Decomposition in the y-z Plane



$\mathbf{z}^T \mathbf{K}\, \mathbf{y} = 0$	Orthonormality
$\lVert\mathbf{z}\rVert_2 = 1$	conditions

The Mathematics of Isolated Bifurcation (1)

We will assume that the first critical point found on the primary path, and characterized by the singular stiffness criterion

$$\boxed{\mathbf{K}(\mathbf{u}_{cr}, \lambda_{cr})\, \mathbf{z}_{cr} = \mathbf{0}}$$

is an isolated bifurcation point.

Therefore the normalized null eigenvector (buckling mode) z: not null , $||\mathbf{z}|| = 1$ is orthogonal to the incremental load vector

$$\boxed{\mathbf{q}^T \mathbf{z} = \mathbf{z}^T \mathbf{q} = 0 \qquad (\mathbf{z} = \mathbf{z}_{cr} \ \text{ for brevity})}$$

Assume that we have located B **and computed z. Our task is to examine the behavior of the system in the vicinity (neighborhood) of** B**.**

In this Chapter we shall be content with looking at the so-called level-one information: tangents to the equilibrium branches that emanate from B

The Mathematics of Isolated Bifurcation (2)

To carry out the task we borrow from algebraic ODE theory. the variation of the state vector measured from its value $\mathbf{u}_B$ at bifurcation is

$$\Delta \mathbf{u} = \mathbf{u} - \mathbf{u}_B$$

Divide this increment by Δt**,** t **being the pseudotime parameter, and pass to the limit** t **-> 0, where the clock starts at** B**:**

$$\dot{\mathbf{u}} = \lim_{t \to 0} \frac{\Delta \mathbf{u}}{\Delta t}$$

The Mathematics of Isolated Bifurcation (3)

The variation rate u from bifurcation can be decomposed into a homogeneous solution component σ z in the buckling mode direction, and a particular solution component y, which is orthogonal to z and goes along the incremental velocity vector:

$$\dot{\mathbf{u}} = (\mathbf{y} + \sigma\, \mathbf{z})\, \dot{\lambda} \qquad \mathbf{y}^T \mathbf{K}\, \mathbf{z} = \mathbf{z}^T \mathbf{K}\, \mathbf{y} = 0$$

The particular solution solves the system

$$\mathbf{K}\, \mathbf{y} = \mathbf{q} \qquad \mathbf{y}^T\, \mathbf{z} = 0$$

which is simply the first-order rate equation $\mathbf{K}\, \dot{\mathbf{u}} = \mathbf{q}\, \dot{\lambda}$ augmented by a normality constraint. Imposing that constraint removes the singularity (rank deficiency) of K at bifurcation point. The geometric interpretation of this decomposition on the y,z plane has been shown on a previous slide.

Linearized Prebuckling Assumptions (1)

We can now restate the LPB assumptions in more precise form.

(I) The external loading is conservative and proportional:

$$\boxed{\mathbf{f} = \mathbf{q}_0 + \lambda\ \mathbf{q}}$$

while the structure is linearly elastic. This implies that the total residual equations are derivable from a potential energy function

(II) The displacements and displacement gradients prior to the critical state are negligible in the sense that (a) the material stiffness matrix can be evaluated in the reference configuration, and (b) the geometric stiffness matrix is proportional to the control parameter λ,

$$\boxed{\mathbf{K}_M \equiv \mathbf{K}_0 \qquad \mathbf{K}_G \equiv \lambda \mathbf{K}_1}$$

in which $\mathbf{K}_1$ is the geometric stiffness for $\lambda = 1$, also evaluated at the reference configuration

Linearized Prebuckling Assumptions (3)

$\mathbf{K}_0$ and $\mathbf{K}_1$ receive the names linear stiffness and reference geometric stiffness, respectively. As discussed in the previous Chapter, the singular stiffness criterion leads to the LPB eigenproblem

$$\boxed{(\mathbf{K}_0 + \lambda \mathbf{K}_1)\,\mathbf{z} = \mathbf{0}}$$

(III) The particular solution y defined previously is obtained by solving

$$\boxed{(\mathbf{K}_0 + \lambda_{cr} \mathbf{K}_1)\,\mathbf{y} = \mathbf{q}}$$

under the orthogonality constraint $\mathbf{y}^T\mathbf{z} = 0$.

Observe that from assumption (I), q is constant, while from assumption (II), both $\mathbf{K}_0$ and $\mathbf{K}_1$ are constant. Hence the LPB eigenproblem is linear in the control parameter λ

Linearized Prebuckling Assumptions (4)

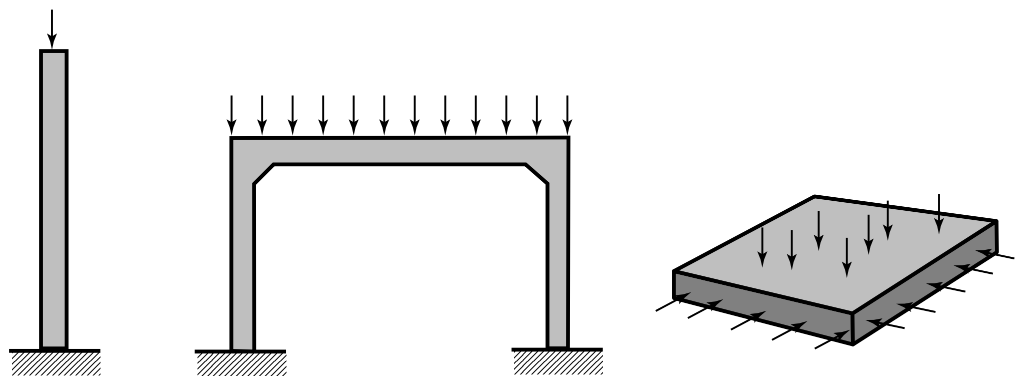
We now prove that if these assmption hold, all critical points determined from the LPB eigenproblem are bifurcation points, that is, $\mathbf{z}^T\,\mathbf{q}$ vanishes. To show that, premultiply both sides of

$$(\mathbf{K}_0 + \lambda_{cr}\mathbf{K}_1)\,\mathbf{y} = \mathbf{q}$$

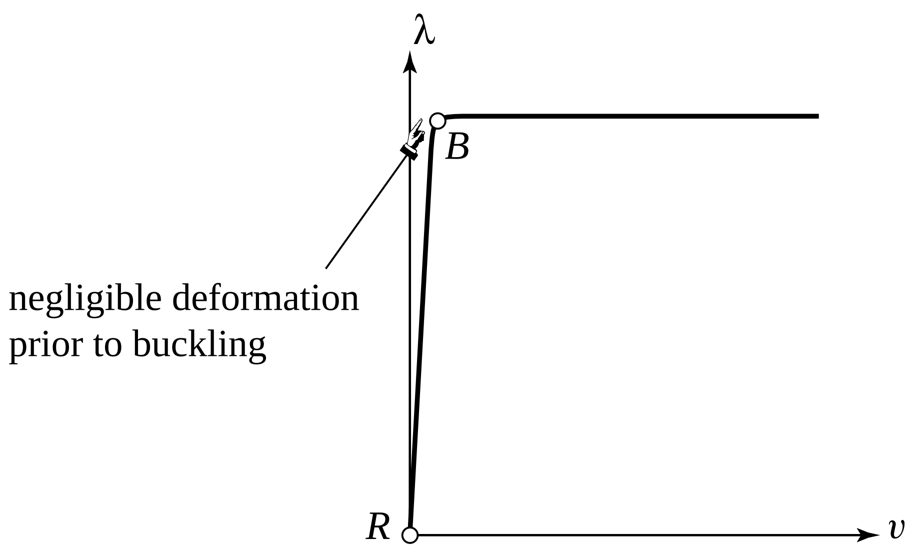
by $\mathbf{z}^T$

$$\mathbf{z}^T\mathbf{q} = \mathbf{z}^T(\mathbf{K}_0 + \lambda\,\mathbf{K}_1)\mathbf{y} = \mathbf{y}^T(\mathbf{K}_0 + \lambda\mathbf{K}_1)\mathbf{z} = \mathbf{y}^T\,(\mathbf{K}\mathbf{z}) = 0$$

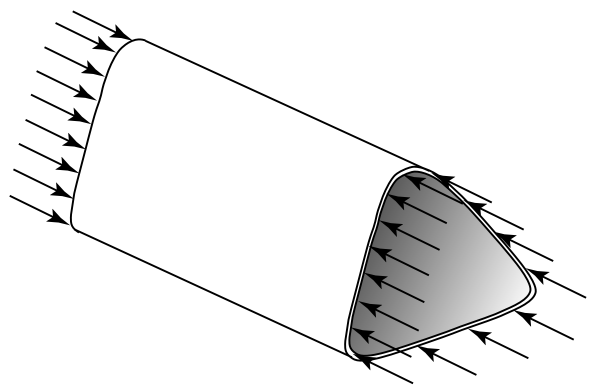
Structures That Fit LPB Assumptions Well



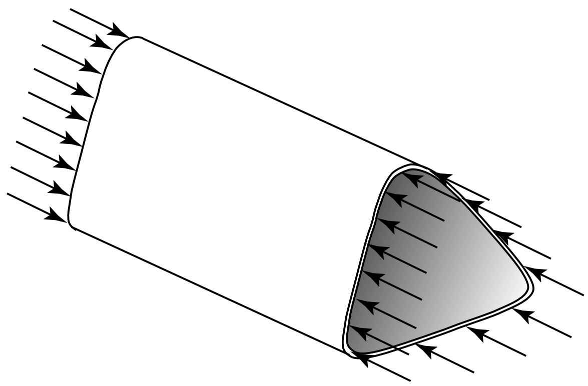
Because Of Their Kind Of Response:



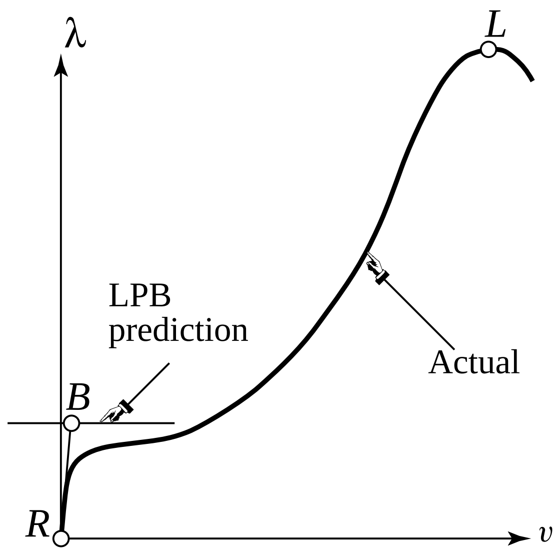
Some Structures That Don't Fit LPB Well (1)



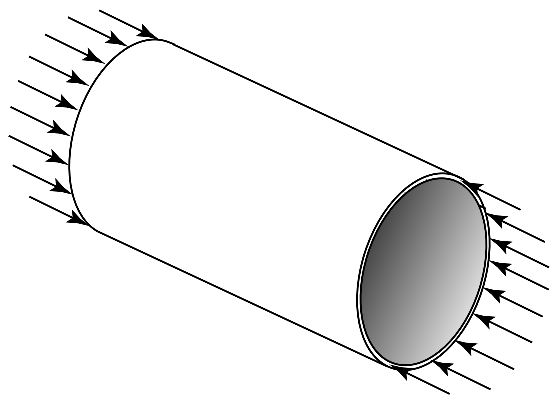
Some Structures That Don't Fit LPB Well
Ex. 1: LPB Grossly Underestimates Strength



Reason: Stress Redistribution Prior to Collapse



Some Structures That Don't Fit LPB Well
Ex. 2: LPB Grossly Overestimates Strength



Reason: Extremely High Imperfection Sensitivity